Calendering of Non-Newtonian Fluids

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Synopsis

The calendering of non-Newtonian fluids by two rotating cylinders to produce thin films of fluids finds wide application in polymer sheet-making and food-drying industries. Theoretical work has previously been devoted to the symmetrical case where the cylinders are of equal diameters rotating at the same speed. The present work proposes a new one-film theory of calendering of power law fluids for unequal radii and surface velocities of the calendering cylinders. The relationship between the dimensionless thickness of the calendered fluid, Δ_e^* and that of the incoming fluid, Δ_i^* is shown to be a function of the ratio of the surface velocities of the cylinders and the power law index. The result further shows that Δ_e^* tends to asymptote after the second decade of Δ_i^*.

INTRODUCTION

Previous work on the calendering of non-Newtonian fluids by Brazinsky et al. and Alston and Astill[1] have always been confined to the symmetrical case where the cylinders are of equal diameters rotating at the same speed. In real applications as in the drum dryer, one often encounters the more difficult unsymmetrical case where the cylinders are of unequal diameters rotating at different speeds. Takserman-Krozer et al.[3] have investigated the unsymmetrical case for Newtonian fluids only. The following theoretical work attempted to solve the unsymmetrical case for power law fluids.

ASSUMPTIONS AND GENERAL EQUATIONS

A non-Newtonian fluid flowing isothermally between two cylinders of radii R_1 and R_2 rotating at the surface velocities of u_1 and u_2, respectively, is considered (refer to Fig. 1).

The following assumptions are made:

1. The fluid can be represented by the power law model of consistency factor K and index n.
2. The width of the clearance between the drum and the applicator roller (the "nip"), 2W, is so small as to be negligible in comparison with length and radii of the cylinders.
3. Since the general movement of the fluid is mainly in the x direction, the velocity of the fluid in the y direction is small.
4. The gradient in the x direction of the velocity in the same direction is negligible compared to its gradient in the y direction.
5. The pressure gradient is a function of x only.

Journal of Applied Polymer Science, Vol. 31, 2457–2465 (1986)
 CCC 0021-8995/86/082457-09$04.00

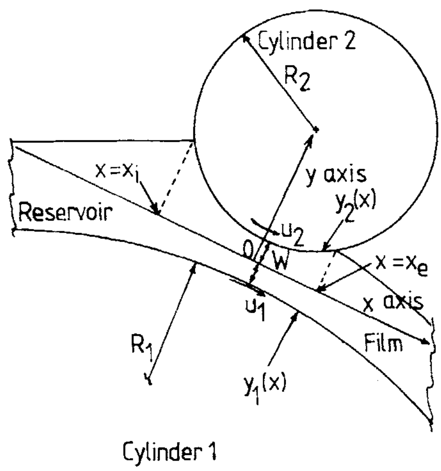

Fig. 1. Calendering of non-Newtonian fluids.

The general equation becomes

$$\frac{dP}{dx} = \frac{d}{dy}(\tau_{xy}) \tag{1}$$

where

$$\tau_{xy} = K\left|\frac{du_x}{dy}\right|^{(n-1)}\frac{du_x}{dy} \tag{2}$$

Where P is the pressure and u_x is the velocity in the x direction.

The boundary conditions are constructed by assuming that the velocity of the fluid at the surfaces of the cylinder is equal to the velocities of the surfaces. The boundary conditions are

$$y = y_1(x),\ u_x = u_1 \tag{3}$$

$$y = y_2(x),\ u_x = u_2 \tag{4}$$

$$x = x_i \quad, P = 0 \tag{5}$$

$$x = x_e \quad, \frac{dP}{dx} = 0, P = 0 \tag{6}$$

where $y_1(x)$ and $y_2(x)$ are the curves representing the surfaces of the cylinders, u_1 and u_2 are the velocities of the surfaces of the cylinders and x_i and x_e are the inlet and exit x ordinate of the nip. To simplify further, the y ordinate of the position of zero velocity gradient $y_m(x)$ is given as

$$y = y_m(x), \frac{du_x}{dy} = 0 \tag{7}$$

The Equations (1)–(7) can be rewritten in dimensionless form.

$$\frac{1}{K^*}\frac{dP^*}{dx^*} = \frac{1}{\chi}\frac{d}{dy^*}\left(\left|\frac{du_x^*}{dy^*}\right|^{(n-1)}\frac{du_x^*}{dy^*}\right) \tag{8}$$

$$y^* = y_1^*(x) \qquad u_x^* = \frac{2}{(1+\nu)} \tag{9}$$

$$y^* = y_2^*(x) \qquad u_x^* = \frac{2\nu}{(1+\nu)} \tag{10}$$

$$x^* = x_i^* \qquad P^* = 0 \tag{11}$$

$$x^* = x_e^* \qquad \frac{dP^*}{dy^*} = 0 \qquad P^* = 0 \tag{12}$$

$$y = y_m^* \qquad \frac{du_x^*}{dy^*} = 0 \qquad u_x^* = u_m^* \tag{13}$$

where the dimensionless variables and parameters are

$$x^{*2} = \chi\frac{2x^2}{W^2} \tag{14}$$

$$y^* = \frac{y}{W} \tag{15}$$

$$u_x^* = \frac{2u_x}{(u_1+u_2)} \tag{16}$$

$$P^* = \frac{P}{P_m} \tag{17}$$

$$K^* = \frac{K(u_1+u_2)}{2P_mW} \tag{18}$$

$$\chi = \sqrt{\frac{(1+\sigma)\Phi}{4}} \tag{19}$$

$$\nu = \frac{u_2}{u_1} \tag{20}$$

$$\sigma = \frac{R_2}{R_1} \tag{21}$$

$$\Phi = \frac{W}{R_2} \tag{22}$$

SOLUTION

Equation (8) is integrated twice with respect to y^* and the boundary condition (13) is applied.

$$u_x^* = u_m^* + \chi^{\frac{1}{n}} \left(\frac{1}{K^*} \frac{dP^*}{dx^*}\right)^{\frac{1}{n}} \frac{n}{(n+1)} (y^* - y_m^*)^{\frac{(n+1)}{n}} \tag{23}$$

For convenience, the y^* variable is transformed to the z^* variable defined by

$$z^* = y^* - y_m^* \tag{24}$$

Equation is rewritten as

$$u_x^* = u_m^* + \chi^{\frac{1}{n}} \left(\frac{1}{K^*} \frac{dP^*}{dx^*}\right)^{\frac{1}{n}} \frac{n}{(n+1)} z^{*\frac{(n+1)}{n}} \tag{25}$$

The boundary conditions (9) and (10) are applied on Eq. (25)

$$\frac{z_2^*}{z_1^*} = \left[\frac{(u_m - u_2)}{(u_m - u_1)}\right]^{\frac{n}{(n+1)}} = \theta \tag{26}$$

The flowrate of fluid through the nip is obtained by integrating Eq. (25) with respect to z^* from one surface of the cylinders to the other.

$$Q^* = u_m^*(z_2^* - z_1^*) + \chi^{\frac{1}{n}} \eta^{\frac{1}{n}} \left(\frac{1}{K^*} \frac{dP^*}{dx^*}\right)^{\frac{1}{n}} \left(z_2^{*\frac{(2n+1)}{n}} - z_1^{*\frac{(2n+1)}{n}}\right) \tag{27}$$

where

$$\eta = \left[\frac{n^2}{(n+1)(2n+1)}\right]^n \tag{28}$$

On applying the boundary conditions (11) and (12) on Eq. (27), the flowrate can be written as

$$Q^* = u_m^* (z_{2e}^* - z_{1e}^*) \tag{29}$$

Substituting Eq. (29) into (27) and rearranging the result gives

$$\frac{1}{K^*}\frac{dP^*}{dx^*}$$

$$= \frac{|u_{me}^*(z_{2e}^* - z_{1e}^*) - u_m^*(z_2^* - z_1^*)|^{(n-1)}[u_{me}^*(z_{2e}^* - z_{1e}^*) - u_m^*(z_2^* - z_1^*)]}{\chi\eta\left[z_2^{*\left[\frac{(2n+1)}{n}\right]} - z_1^{*\left[\frac{(2n+1)}{n}\right]}\right]^n} \tag{30}$$

Equation (30) is integrated with respect to x^* and boundary conditions (11) and (12) are applied

$$\int_{x_i^*}^{x_e^*}\left[\frac{|u_{me}^*(z_{2e}^* - z_{1e}^*) - u_m^*(z_2^* - z_1^*)|^{(n-1)}[u_{me}^*(z_{2e}^* - z_{1e}^*) - u_m^*(z_2^* - z_1^*)]}{\left[z_2^{*\left[\frac{(2n+1)}{n}\right]} - z_1^{*\left[\frac{(2n+1)}{n}\right]}\right]^n}\,dx^*\right] = 0 \tag{31}$$

The surface of the cylinders can be approximated by

$$y_1^*(x^*) = -1 + \frac{2}{(1+\sigma)}x^{*2} \tag{32}$$

$$y_2^*(x^*) = 1 + \frac{2\sigma}{(1+\sigma)}x^{*2} \tag{33}$$

Hence

$$z_2^* - z_1^* = y_2^* = y_1^* = 2(1 + x^{*2}) \tag{34}$$

$$z_1^* = -\frac{2(1 + x^{*2})}{(1+\theta)} \tag{35}$$

$$z_2^* = \frac{2\theta(1 + x^{*2})}{(1+\theta)} \tag{36}$$

Equations (34), (35), and (36) are substituted into Eq. (30).

$$\frac{1}{K^*}\frac{dP^*}{dx^*} = \frac{|Q^* - 2u_m^*(1 + x^*)^2|^{(n-1)}[Q^* - 2u_m^*(1 + x^{*2})]\,(1+\theta)^{(2n+1)}}{\chi\eta\,[2(1 + x^{*2})]^{(2n+1)}\left[1 + \theta^{\left[\frac{(2n+1)}{n}\right]}\right]^n} \tag{37}$$

Equation (37) is then substituted into Eq. (25) which is evaluated at $z^* = z_2^*$. The resulting equation is

$$\frac{Q^*}{2(1 + x^*)} = \frac{[u_1{}^* \theta^{\left[\frac{(n+1)}{n}\right]}(1 - \gamma) - u_2{}^*(\theta^{\left[\frac{(n+1)}{n}\right]} - \gamma)]}{\gamma\left[1 - \theta^{\left[\frac{(n+1)}{n}\right]}\right]} \tag{38}$$

where

$$\gamma = \frac{(2n + 1)\,\theta^{\left[\frac{n+1}{n}\right]}(1 + \theta)}{n\left[1 + \theta^{\left[\frac{(2n+1)}{n}\right]}\right]} \tag{39}$$

It can be shown that

$$\frac{Q^*}{2(1 + x^*)} - u_m{}^* = \frac{1}{(1 - \gamma)}\left[\frac{Q^*}{2(1 + x^{*2})} - u_2{}^*\right] \tag{40}$$

Equation (31) can be rewritten as

$$\int_{x_i{}^*}^{x_e{}^*}\left[\frac{\left|\frac{Q^*}{2(1 + x^{*2})} - u_2{}^*\right|^{(n-1)}\left[\frac{Q^*}{2(1 + x^{*2})} - u_2{}^*\right](1 + \theta)^{(2n+1)}}{(1 - \gamma)^n(1 + x^{*2})^{(n+1)}\left[1 + \theta^{\left[\frac{(2n+1)}{n}\right]}\right]^n}\right]dx^* = 0 \tag{41}$$

The dimensionless thickness of the incoming fluid is given by

$$\Delta_i{}^* = 2(1 + x_i{}^{*2}) \tag{42}$$

The dimensionless thickness of the calendered fluid is given by

$$\Delta_e{}^* = 2(1 + x_e{}^{*2}) \tag{43}$$

RESULTS AND DISCUSSION

A solution involving the determination of θ for given values of Q^* and x^* using Eq. (38) is too tedious. The easier alternative is the determination of the values of $\frac{Q^*}{2(1 + x^{*2})}$ (real values only) for given values of θ. These paired values of $\frac{Q^*}{2(1 + x^{*2})}$ and θ are then used to determine $x_i{}^*$ for any given $x_e{}^*$ such that the integral 41 is zero. The results are plotted in Figure 2 and 3.

These results indicate clearly that by expressing x^* as in Eq. (14), the dimensionless thickness of the calendered sheet, $\Delta_e{}^*$ is apparently independent of the ratio of the radii of the cylinders σ and the nip width factor ϕ. This result is not surprising because the expression in Eq. (14) actually transforms the asymmetrical case to an equivalent symetrical problem. The relationship between $\Delta_e{}^*$ and $\Delta_i{}^*$ is shown to be a function of the ratio of the velocities of the surfaces of the cylinders ν and the power law index of the fluid n.

Figure 2 shows that for small $\Delta_i{}^*$ (less than 10), $\Delta_e{}^*$ increases as ν increases. However for $\Delta_i{}^*$ larger than 10, $\Delta_e{}^*$ decreases as ν increases. Figure 3 on the other hand indicates that $\Delta_e{}^*$ decreases with increasing n. $\Delta_e{}^*$ tends to asymptotes after the second decade of $\Delta_i{}^*$. This last result is consistent with the result of Brazinsky et al.[3] for the symmetrical case and that of Takserman-Krozer et al.[3] for the unsymmetrical case with Newtonian fluids. In practical terms, it means that any increase in the thickness of the incoming fluid after the second decade of $\Delta_i{}^*$ does not increase the thickness of the calendered film by any significant amount. In other words, excessive flooding of the entrance of the nip does not increase the thickness of the film.

CONCLUSION

The dimensionless thickness of the calendered film Δ_e^* was found theoretically to depend explicitly on the power law index n of the fluid, the relative velocities of the cylinders ν and the incoming dimensionless thickness of the fluid $\Delta_i{}^*$. $\Delta_e{}^*$ is apparently independent of the ratio of the radii of the cylinders σ only because the dimensionless space variable x^* has

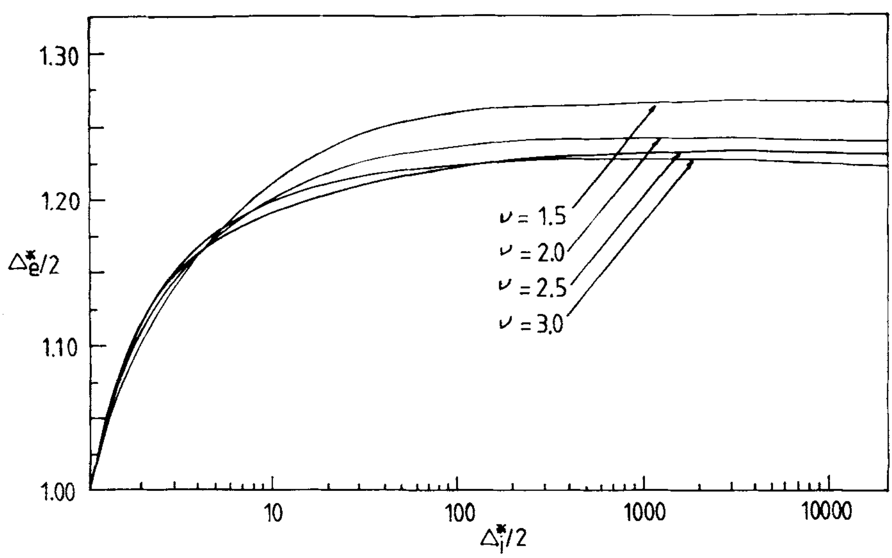

Fig. 2. Theoretical curves of $\Delta_e{}^*/2$ versus $\Delta_i{}^*/2$ for n = 0.6 and various values of ν.

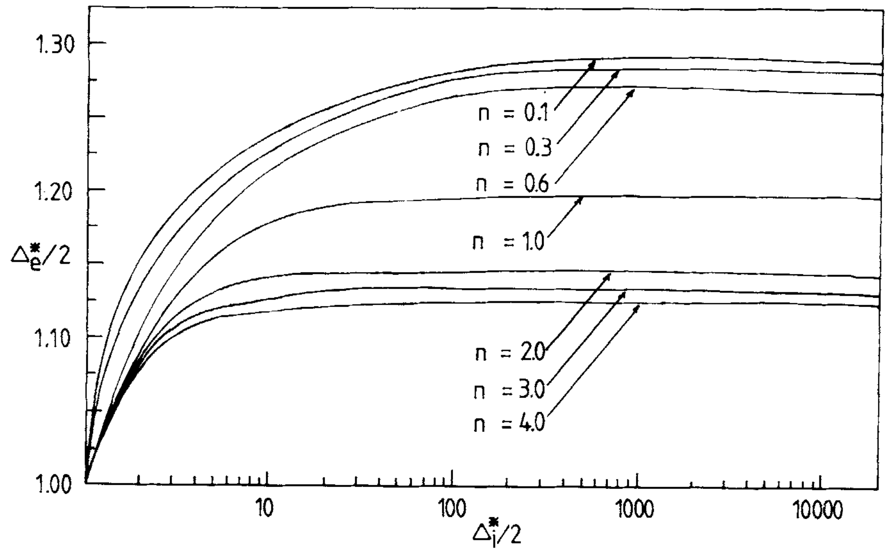

Fig. 3. Theoretical curves of $\Delta_e{}^*/2$ versus $\Delta_i{}^*/2$ for $\nu = 1.5$ and various values of n.

been transformed to include σ within it. $\Delta_e{}^*$ approaches an asymptote after the second decade of $\Delta_i{}^*$.

NOMENCLATURE

Latin Symbols

K	Consistency coefficient of a power law fluid
K^*	Dimensionless coefficient of the consistency coefficient of a power law fluid
n	Index for power law fluid
N	Speed of rotation of the drum
P	Pressure
P^*	Dimensionless pressure
P_m	Maximum pressure
Q	Volumetric flow rate
Q^*	Dimensionless flow rate
R_1	Radius of cylinder 1
R_2	Radius of cylinder 2
u_1	Velocity of the surface of cylinder 1
u_a	Velocity of air near the surface of the drum
u_m	Maximum velocity
$u_m{}^*$	Dimensionless maximum velocity
u_{me}	Maximum velocity at the exit of nip
u_x	Velocity in the x direction
$u_x{}^*$	Dimensionless velocity in the x direction
W	Half width of nip
x	Space variable
x^*	Dimensionless space variable
x_e	Value of space variable x at the exit of the nip

x_e^*	Dimensionless x_e
x_i	Value of space variable x at the inlet of the nip
x_i^*	Dimensionless x_i
y	Space variable
y^*	Dimensionless space variable
$y_1(x)$	Curve representing the surface of cylinder 1
$y_1^*(x)$	Dimensionless $y_1(x)$
$y_2(x)$	Curve representing the surface of cylinder 2
$y_2^*(x)$	Dimensionless $y_2(x)$
$y_m(x)$	Value of space variable y at the point of maximum velocity
$y_m^*(x)$	Dimensionless $y_m(x)$
z	Space variable
z^*	Dimensionless z
$z_1(x)$	Curve representing the surface of cylinder 1
$z_1^*(x)$	Dimensionless $z_1(x)$
$z_2(x)$	Curve representing the surface of cylinder
$z_2^*(x)$	Dimensionless $z_2(x)$

Greek Symbols

γ	Dimensionless coefficient involving n and θ
η	Dimensionless coefficient involving the power law index n
θ	The ratio of the distance between the surface of cylinder 2 and the point of maximum velocity to that between the surface of cylinder 1 and the point of maximum velocity
ν	Ratio of the velocity of the surface of cylinder 2 to that of cylinder 1
σ	The ratio of the radius of cylinder 2 to that of cylinder 1
τ_{xy}	Shear stress
ϕ	Ratio of the half width of nip to the radius of cylinder 2
χ	Dimensionless coefficient
Δ_e^*	Dimensionless thickness of the calendered fluid
Δ_i^*	Dimensionless thickness of the incoming fluid

References

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Received February 27, 1985
Accepted April 27, 1985